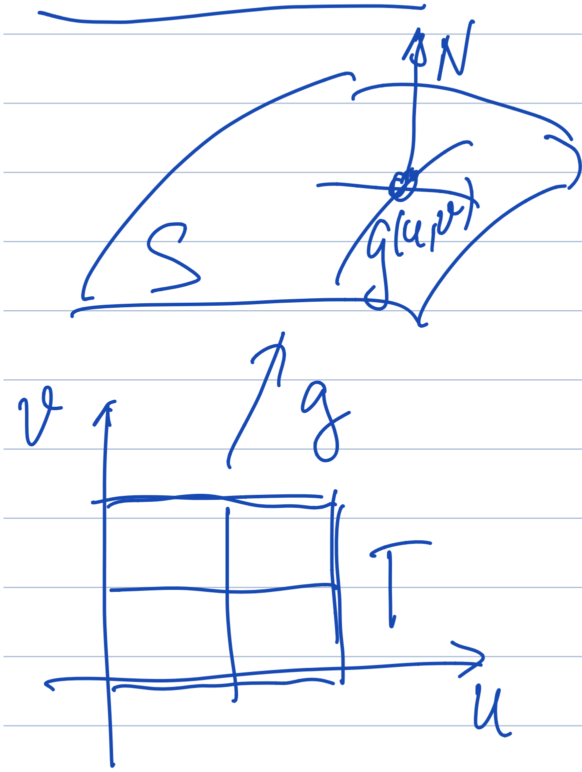
CDI-II — Prática 2/6/21

Ficha 12 + Ficha 13

Ficha 12 : Fluxo, T. Divergência



$$\int_S F\cdot N = \iint_T F(g(u,v))\cdot D_u g \times D_v g \; du\,dv$$

Ficha 12:

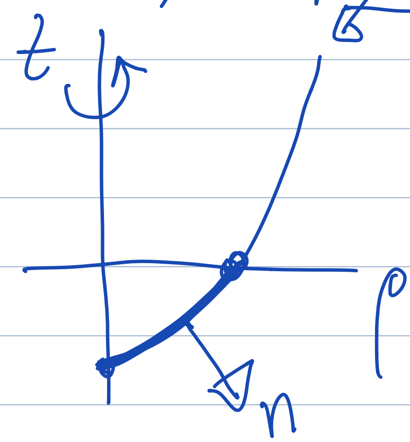
4 — dados: A, $n_z < 0$, H

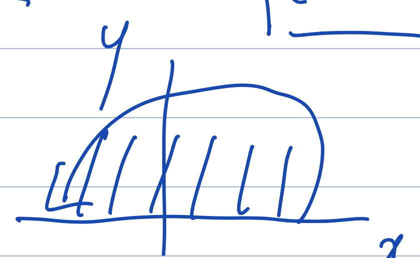
$$\int_A H \cdot n = ?$$

Parametrizar A:

$z = x^2 + y^2 - 1$, $z < 0$, $y > 0$

(ρ, θ, z) $\quad \boxed{z = \rho^2 - 1} < 0 \iff \boxed{\rho < 1}$





$$\boxed{0 < \theta < \pi}$$

$$g(\rho,\theta) = (\rho\cos\theta, \rho\sin\theta, \rho^2-1)$$

$$T\begin{cases} 0<\rho<1 \\ 0<\theta<\pi \end{cases}$$

$$\begin{array}{ccc} i & j & k \end{array}$$

$$D_\rho g = (\cos\theta, \sin\theta, 2\rho)$$

$$D_\theta g = (-\rho\sin\theta, \rho\cos\theta, 0)$$

tangentes

$$-D_\rho g \times D_\theta g = (+2\rho^2\cos\theta, +2\rho^2\sin\theta, -\rho) \quad \text{Normal}$$

$n_z < 0$

$$H(g(\rho,\theta)) = (-\rho\sin\theta, \rho\cos\theta, \rho^2-1)$$

$$-H(g(\rho,\theta)) \cdot D_\rho g \times D_\theta g = -\rho(\rho^2-1)$$

$$\int_A H\cdot n = -\int_0^{\pi}\left(\int_0^1 \rho(\rho^2-1)\,d\rho\right)d\theta$$

etc ...

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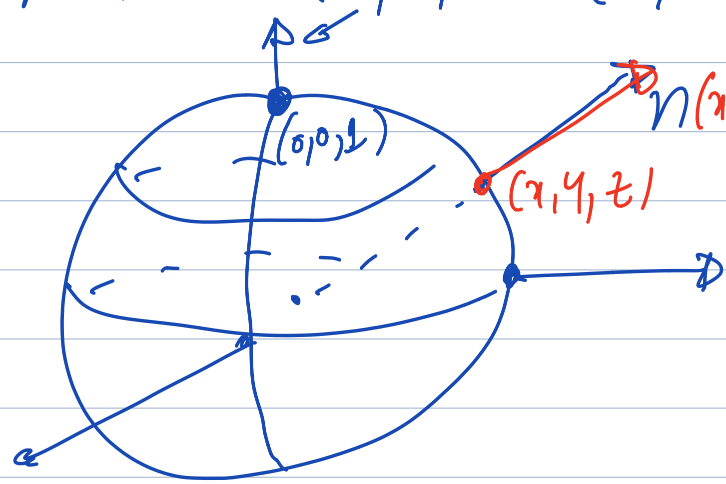
6 — $F(x,y,z) = h(r)(x,y,z)$

$r = \|(x,y,z)\|$

S : $\boxed{x^2+y^2+z^2=1}$

n : $n(0,0,1) = (0,0,1)$



$n(x,y,z) = (x,y,z)$

$$\int_S F\cdot n \equiv \iint_S \underbrace{F\cdot n}$$

Se $F\cdot n = c \in \mathbb{R}$, então

$$\int_S F\cdot n = c \operatorname{vol}_2(S).$$

S: $x^2+y^2+z^2=1$ $\quad \boxed{r=1}$

$$n = \frac{(2x, 2y, 2z)}{\sqrt{4x^2+4y^2+4z^2}} = (x,y,z)$$

$$F(x,y,z) = h(1)(x,y,z)$$

$$n(x,y,z) = (x,y,z)$$

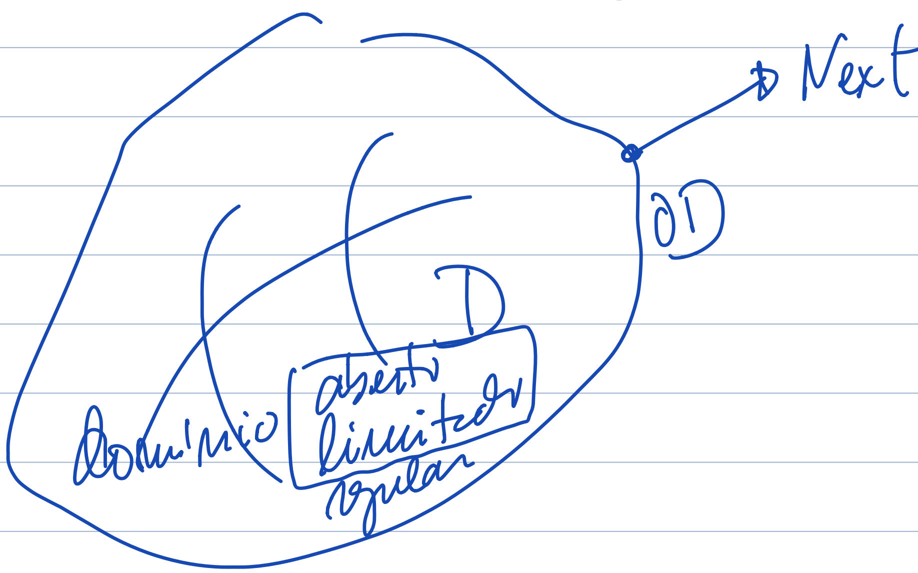
$$F\cdot n = h(1)(x^2+y^2+z^2) = h(1)$$

$$\int_S F\cdot n = h(1)\operatorname{vol}_2(S) = 4\pi h(1) \;//$$

T. Divergência ($\mathbb{R}^3$)

$$\iiint_D \operatorname{div} F = \iint_{\partial D} F \cdot N_{ext}$$

Fluxo



Se $\operatorname{div} F = 1$, $\displaystyle\iiint_D \operatorname{div} F = vol_3(D)$

?

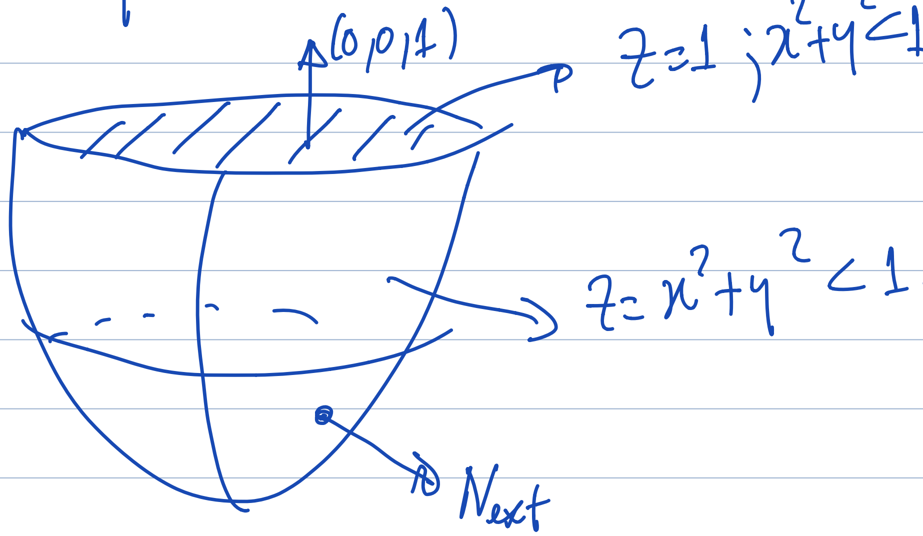
$$\boxed{vol_3(D)} = \iint_{\partial D} F \cdot N_{ext} \rightarrow \text{calcular}$$

$F(x,y,z) = (x,0,0) \to \operatorname{div} F = 1$

$F(x,y,z) = (0,y,0) \to \operatorname{div} F = 1$

$F(x,y,z) = (0,0,z) \to \operatorname{div} F = 1$

$D: \; x^2+y^2 < z < 1$



Usar $F(x,y,z) = (x,0,0)$.

etc.

9 – <u>Dados</u>: S, $n_z > 0$, F

$$\int_S F \cdot n = \text{?} \quad \left(\text{c/ } \underline{\text{T. da divergência}}\right)$$

$$\iint_{\partial D} F \cdot n_{ext} = \iiint_D \operatorname{div} F$$

De S construir D tal que

$$S \subseteq \partial D$$

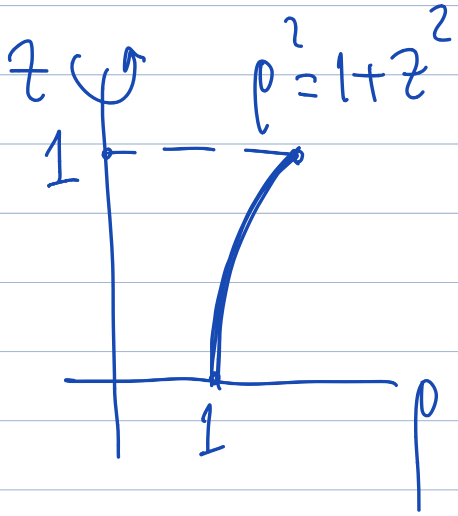
$$S \longrightarrow D$$

$=$ 1 eq. $\qquad$ <u>inequação</u>

$$S \longrightarrow D \ \text{aberto}, \ \boxed{\text{limitado}}$$

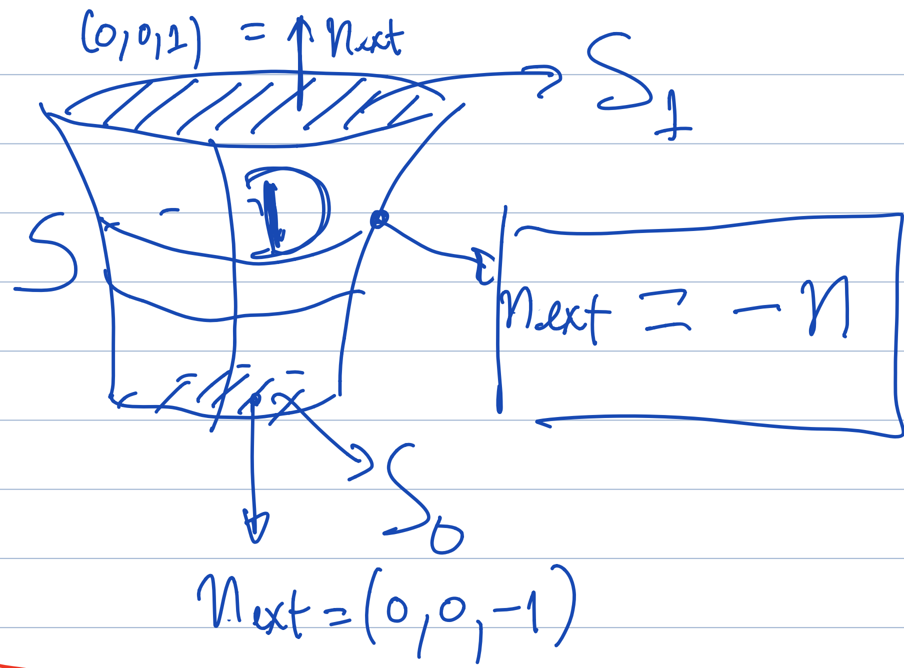
$$= \longrightarrow > \text{ ou } <$$

$$\begin{cases} x^2+y^2=1+z^2 \\ 0<z<1 \end{cases} \longrightarrow D \begin{cases} x^2+y^2<1+z^2 \\ 0<z<1 \end{cases}$$



$$\partial D = S \cup \underbrace{\begin{Bmatrix} z=0 \\ x^2+y^2<1 \end{Bmatrix}}_{S_0} \cup \underbrace{\begin{Bmatrix} z=1 \\ x^2+y^2<2 \end{Bmatrix}}_{S_1}$$

$$\partial D = S \cup S_0 \cup S_1$$



$$\boxed{\iint_S F\cdot n_{ext}} + \iint_{S_0} F\cdot n_{ext} + \iint_{S_1} F\cdot n_{ext} = \iiint_D \operatorname{div} F$$

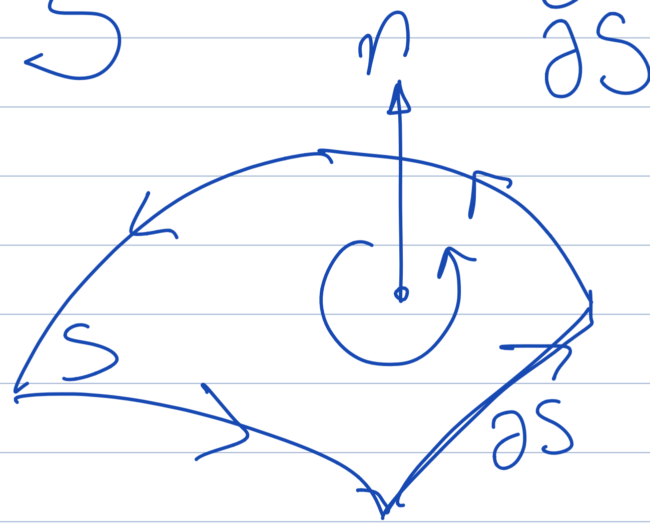
?

Calcular !!!

$$\iint_S F\cdot n = -\iint_S F\cdot n_{ext} \qquad \text{etc.} \ldots$$

T. Stokes ($\mathbb{R}^3$)

$$\iint_S \text{rot}\, F \cdot n = \oint_{\partial S} F \cdot d\gamma$$



$$\boxed{\text{rot}\, F = \nabla \times F}$$

1 — Dados: F, S, n

Questão: $\displaystyle\int_S \text{rot}\, F \cdot n = \,?$

Solução: $\displaystyle\oint_{\partial S} F \cdot d\gamma$ (calcular)

$S^{(\text{Superfície})} \longrightarrow \partial S^{(\text{linha})}$

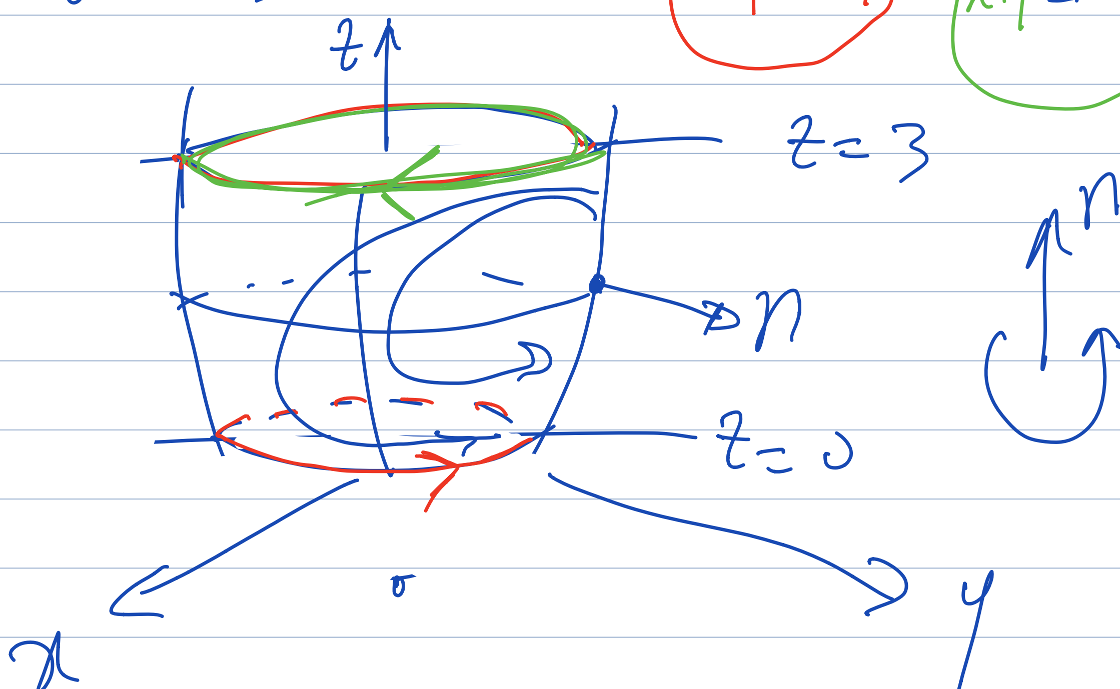
$\supseteq$ $\qquad\qquad$ $\equiv$

$\subseteq$ $\qquad\qquad$ $\supset$

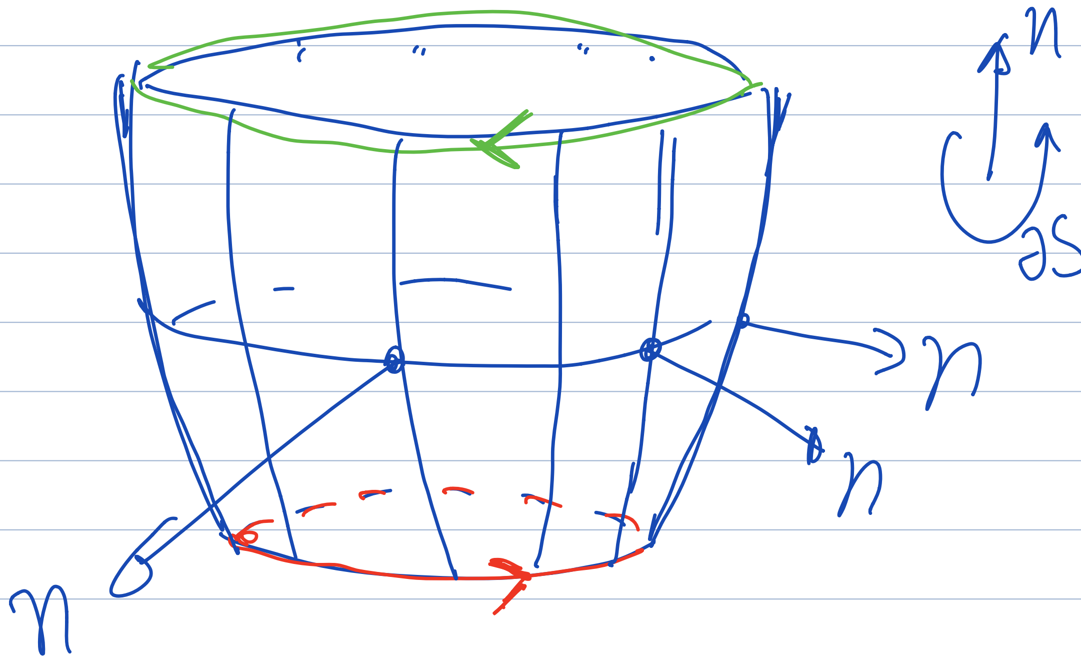
$\leq$

$z = x^2+y^2-1$ $\longrightarrow$ $\boxed{\begin{array}{c} z=0 \\ x^2+y^2=1 \end{array}} \cup \boxed{\begin{array}{c} z=3 \\ x^2+y^2=4 \end{array}}$

$0 < z < 3$



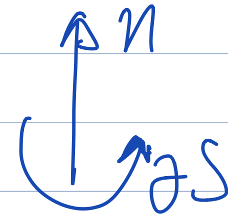
etc.



3 - Dados: H, L, g

$$\int_L H \cdot dg = ?$$

Stokes:



$$\oint_{\partial S} F\cdot dg = \iint_S \mathrm{rot}\, F\cdot n$$

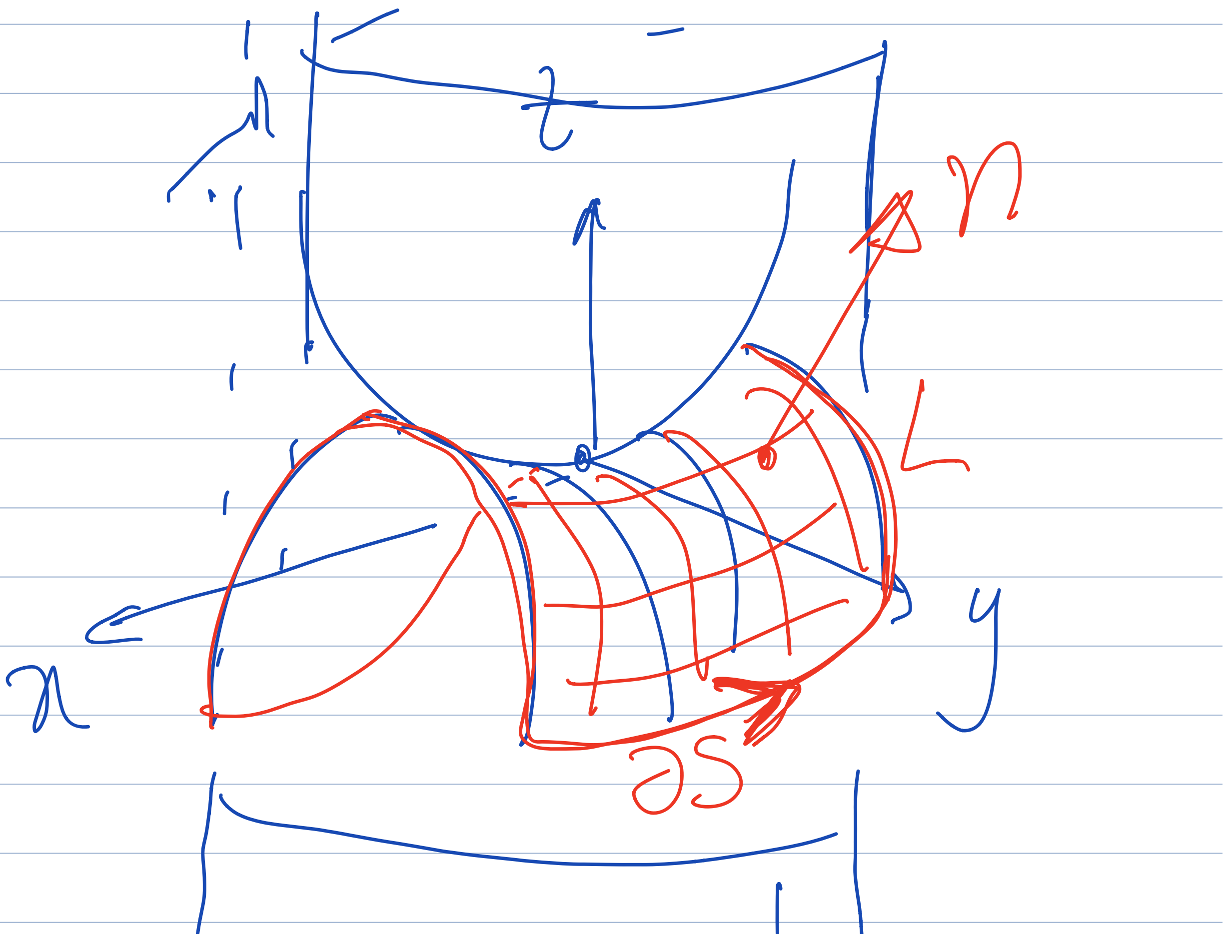
Solução: de L construir S tal que $L = \partial S$.

$$L \longrightarrow S \quad \underline{\text{limitada}}$$

$$g(t) = (\cos t, \operatorname{sen} t, \cos 2t) \Longrightarrow \boxed{\begin{array}{l} x^2+y^2=1 \\ z = x^2-y^2 \end{array}}$$

$$\begin{cases} x^2+y^2=1 \\ z=x^2-y^2 \end{cases} \longrightarrow \begin{array}{c} x^2+y^2<1 \\ \boxed{z=x^2-y^2} \end{array}$$

$$L \longrightarrow S \text{ limitado}$$



$$\boxed{\partial S = L}$$

$$\int_L F\cdot dq = \int_{\partial S} F\cdot dq = \iint_S \operatorname{rot} F \cdot n$$

etc.